Parameter Estimation Using Dual Fractional Power Filters

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ABSTRACT

Many correlation filters have been designed to be invariant to certain parameters within a set of input images. For example, the construction of rotation and scale invariant filters is well documented. However, an estimation of a generic varying parameter is not available by these methods. This paper presents a two-filter method that estimates the value of a varying parameter in the input image.

KEYWORDS : Fractional Power Filter, Synthetic Discriminant Functions, Parameter Estimation

1. INTRODUCTION

This paper will introduce a dual filter system that will estimate the magnitude of a varying parameter for a set of input images. The set of images **X** = { X_1, X_2, ..., X_N} is created by varying a parameter q in a single image I, where q is the varying parameter or parameters (e.g., rotation, scale, etc.). In fact, q does not have to be defined by the user. It is the parameter or parameters that vary within the training set. An inherent advantage of the class of filters used here is that the varying parameters do not have to be defined by the user, they only have to be contained within the training set. A dual filter system will be created from a subset of **X** that will be capable of recognizing all X_i within **X** and estimate q for any X_i.

Previous work related to this filter system have included many methods to create a q-invariant, single filter recognition system.[1-8] Many of these types of systems require a q-specific transformation of the input image before the filter is applied. An approach that creates a filter invariant to a general q is the synthetic discriminant functions (SDF) which are reviewed in ref. 9. Unlike the previous methods, the SDF class of filters doe not require that a particular q be defined. This class of filter can be invariant to any q inherent in a training set.

SDFs have been used to estimate q for training images.[10] This estimation was obtained by associating each image with a different value (established in the constraint vector). When an unknown input from the training set was operated on by the filter the constraint value associated with that input was produced, thus indicating which training input was the actual input. This method only works on the training inputs and not the intermediate cases.

The method used here will use two filters which are members of the SDF class. These filters are Fractional Power Filters (FPFs) which will be reviewed in Section 2. Section 3 will present the method by which the FPFs can be used to estimate q. Section 4 will present three examples.

2. REVIEW OF THE FRACTIONAL POWER FILTER

The Fractional Power Filter (FPF)[11] is a superset of two standard SDF-class filters: the SDF and the MACE filter. This section will review the SDF, MACE and FPF filters. Again, further understanding of the SDF, the MACE, and other variants of this class of filter is available in Kumar (ref 9).

2.1. Synthetic Discriminant Function

The SDF is a weighted linear combination of inputs. For this discussion the inputs are vectors of dimension K. Given a set of N column input vectors v_i (i = 1,2,...,N) a matrix $\hat{\mathbf{V}}$ is created by the combination of the Fourier transforms of the input vectors as in,

$$\hat{\mathbf{V}} = \left[\hat{v}_1 \middle| \ \hat{v}_2 \middle| \ \ldots \middle| \ \hat{v}_N \right] \tag{1}$$

where $\hat{v}_i$ is the Fourier transform of v_i.

Each vector $\hat{v}_i$ is associated with a constraint value c_i. Thus, the SDF filter, **h**, is constrained by,

$$\hat{\mathbf{V}}\,\hat{\mathbf{h}} = \mathbf{c} \quad . \tag{2}$$

Since $\hat{\mathbf{V}}$ is not square, the pseudo-inverse is used to find **h**,

$$\hat{\mathbf{h}} = \hat{\mathbf{V}}\left[\hat{\mathbf{V}}^T\hat{\mathbf{V}}\right]^{-1}\mathbf{c} \,. \tag{3}$$

2.2. Minimum Average Correlation Energy Filter

The Minimum Average Correlation Energy (MACE) filter is similar to the SDF but it has the additional condition of minimizing the correlation surface energy. The MACE filter will only be presented here, however the derivation is presented in Mahalanobis (ref 12). The MACE filter is computed by,

$$\hat{\mathbf{h}} = \mathbf{D}^{-1}\hat{\mathbf{V}}\left[\hat{\mathbf{V}}^T\mathbf{D}^{-1}\hat{\mathbf{V}}\right]^{-1}\mathbf{c} \,, \tag{4}$$

where

$$D_{ij} \ = \ \frac{d_{ij}}{N}\sum_k |\,\hat{v}_{k,i}\,|^2 \quad . \tag{5}$$

where $\hat{v}_{k,i}$ is the i-th element of the k-th vector.

2.3. The Fractional Power Filter

Consider the power term in Eq. 5. If this term were 0 then **D** would become the Identity. In this scenario the SDF could be computed by Eq. 4. The only difference between the SDF and MACE is the power term in Eq. 5. The FPF allows this power term to vary between 0 and 2. Thus, the FPF is computed by Eq. 5 where,

$$D_{ij} \ = \ \frac{d_{ij}}{N}\sum_k |\,\hat{v}_{k,i}\,|^p, \ \ p \ = \ [0,2] \tag{6}$$

Values of $p<0$ and $p>2$ do not seem to alter the performance from the $p=0$ and $p=2$ cases respectively. Values of $p<0$ would enhance the stronger frequencies and provide no real benefit to the filter performance. At $p=2$ the energy of the correlation surface is minimized. The energy of this surface cannot be lowered by values of $p > 2$. Thus, the values of p are kept between 0 and 2 inclusive.

2.4. The Continuum

The important information is the value of output peak of the correlation surface. Consider a filter constructed from a set of training vectors with a varying parameter q. For the training vectors the constrained value will be the peak value (except for some cases where $p \approx 0$). For the values of q between the training q's the output peak value will be less than the constrained value. This is shown in Kumar (ref. 9) and specifically for the FPF in Brasher (ref. 11). For an ideal case this behavior is shown in Fig. 1. This behavior will be used to build a two filter system. For some cases for $p \approx 0$ the output values near the constrained value can be actually higher than the constrained value. For this reason the dual system tends to avoid the use of very small q's.

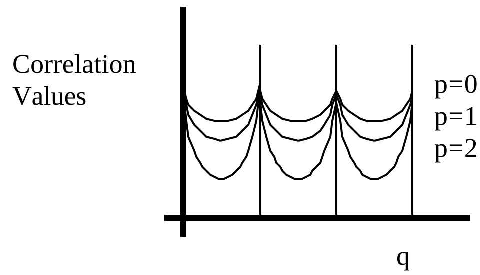

Figure 1. The typical behavior of the FPF filter. For differing p's the peak value is 1 for the training q's and lower for intermediate q's. A larger p produces more droop in the curves.

The ideal behavior shown in Fig. 1 shows peaks in the curve which are the correlation peak values for the training q's. The user defines the range of q's, the distance between the training q's (by selecting the training set), the response of each training q (by selecting the constraint vector), and the curvature of the continuum between the training q's (by selecting a p value).

3. ESTIMATION OF THE VARYING PARAMETER

This section will present a method by which q can be estimated for a non-training input image. This method will use two (or more in some cases) FPFs and take advantage of the continuum behavior shown in Section 2.4.

3.1. 2-D Inputs

The first order of business is to convert the FPF equations from 1-D inputs and a 1-D filter to 2-D inputs and a 2-D filter by creating a tensor **X** which is created by a set of training images similar to Eq. 1. The filter is created by modifying Eqs. 4 and 6 to produce,

$$\hat{\mathbf{h}} = \mathbf{D}^{-1/2}\hat{\mathbf{Y}}\left[\hat{\mathbf{Y}}^T\hat{\mathbf{Y}}\right]^{-1}\mathbf{c}\,, \tag{7}$$

where

$$\hat{\mathbf{Y}} \equiv \mathbf{D}^{-1/2}\hat{\mathbf{X}}\,. \tag{8}$$

The elements of $\hat{\mathbf{Y}}^T\hat{\mathbf{Y}}$ are the inner products of the images

$$\hat{\mathbf{Y}}^T\mathbf{Y} = \begin{bmatrix} \hat{\mathbf{Y}}_0^+ \cdot \hat{\mathbf{Y}}_0 & \hat{\mathbf{Y}}_0^+ \cdot \hat{\mathbf{Y}}_1 & \cdots & \hat{\mathbf{Y}}_0^+ \cdot \hat{\mathbf{Y}}_N \\ \vdots & & \ddots & \vdots \\ \hat{\mathbf{Y}}_N^+ \cdot \hat{\mathbf{Y}}_0 & & \cdots & \hat{\mathbf{Y}}_N^+ \cdot \hat{\mathbf{Y}}_N \end{bmatrix}, \qquad (9)$$

where $\hat{\mathbf{Y}}_k^+$ is the complex conjugate of $\hat{\mathbf{Y}}_k$, and

$$D_{ijkl} = \frac{d_{ijkl}}{N} \sum_n |\hat{x}_{n,ij}|^p \ . \qquad (10)$$

3.2. Creation of the Possibility Sets

Given an input with an unknown value of q the response of the FPF will lie somewhere on the curve in Fig. 1. However, there exist several possible inputs that create the same output. Thus, a single peak value creates a set of possible q values. This is graphically displayed in Fig. 2. This set of possible q values is denoted by Q = { q_1, q_2, ..., q_M }.

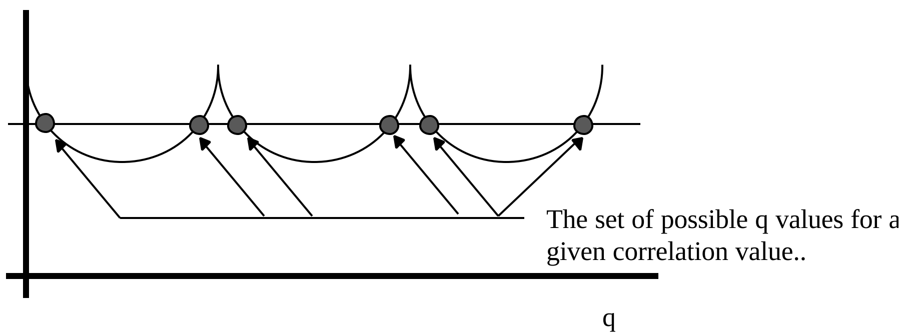

Figure 2. The determination of the set Q is all possible q values for a given peak correlation value. In this case, there are 6 members of the set Q.

3.3. Parameter Estimation

The creation of one filter will lead to the creation of one possibility set Q_1. By altering training parameters (set of training inputs, p, and **c**) a second filter can be created that will have a possibility set Q_2. The q of the input image must be in both Q_1 and Q_2. However, proper construction of the two filters will prevent Q_1 and Q_2 from containing any other common elements. The thrust of this algorithm is to create two FPFs such that $Q_1 \cap Q_2$ 1 is a set with only one member. That member is the estimation of the q of the input. In actuality, each member of the Q's is a small range of q values to compensate for noise, errors, and other destructive effects.

Proper construction of the two filters is dependent upon the type of input data. For a general case, proper construction of the two filters would require that one or more of the training parameters be radically different in the construction of the two filters. For the cases when this is not feasible (limited training range, limited output resolution due to noise, etc.) more than two filters may need to be used. The goal in a multi-filter system is still the same: to create a set from $Q_1 \cap Q_2 \cap \ldots \cap Q_Z$ 2 with only one member. That one member is the estimation of q.

4. EXAMPLE

Three examples are presented in this section. The first considers noiseless images that varying in rotation. The second example considers noiseless images that vary in scale. The third example considers a data set that varies in scale and rotation (coupled) and with three different levels of random noise.

4.1. Rotation

This example built two FPFs from a the Fourier transform of rotated versions of the image in Fig. 3 of a white blood cell surrounded by several red blood cells. Thus, q is rotation. The filters were built with the following parameters:

Filter 1: N = 3 q_i = 0°, 27°, 54°
p = 0.5 c = { 1, 1, 1 }
Filter 2: N = 6 q_i = 0°, 12°, 24°, 26°, 48°, 60°
p = 1.3 c = { 1, 1, 1, 1, 1, 1 }

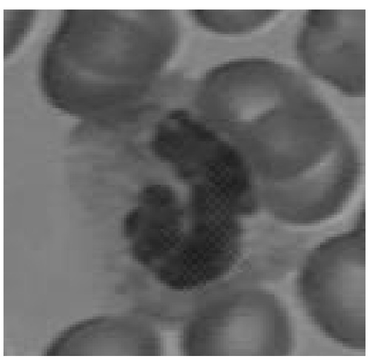

Figure 3. An input image of a white blood cell surrounded by several red blood cells.

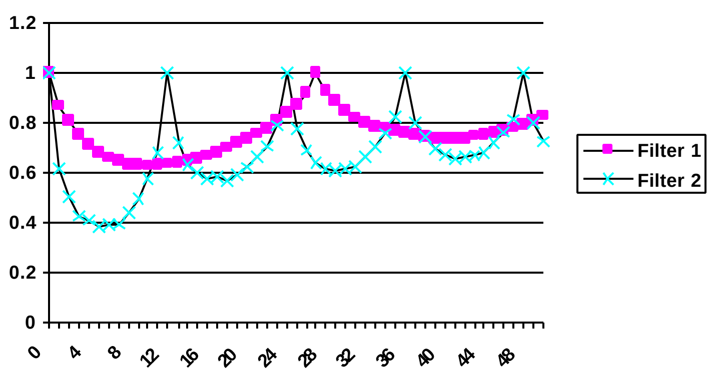

Figure 4. A graph of the continuum for the rotation example case. Measurements were taken in increments of one degree of rotation.

The continuum for this image is shown in Fig. 4. Measurements were taken for 1 degree of rotation. The fact that the curve is extremely smooth indicates that extrapolation to non-measured points is very possible.

A test of this algorithm is to rotate the input by some random q within the range used for the construction of the filters. The output of the two filters each provide a set of possible rotations. The intersection of the two set should have one element which indicates the rotation of the original image.

One case is shown as an example. The rotation which was randomly selected was q = 43.1 degrees. The output of the two filters were 0.746 (Filter 1) and 0.668 (Filter 2). The output of the filter will not exactly match any of the continuum outputs since it is not exactly one of the rotations. So each member of Q has a range (denoted by [ ]) and the union of the two sets searches for a common range. The Q sets are

Filter 1 Q = { [3,4], [20,21], [37,38], [43,44] }
Filter 2 Q = { [0,1], [11,12], [12,13], [21,22], [26,27], [32,33], [39,40], [43,44] }.

The union of the two sets produces only one member [43,44]. This range is examined more closely. The peak measurements that are shown in the continuum and the unknown input are shown in the Table 1.

q	Filter 1	Filter 2
43	0.7457	0.6673
unknown	0.746	0.668
44	0.7535	0.6793

By linear extrapolation the unknown is estimated to be 43.04 (Filter 1) and 43.05 (Filter 2). The correct rotation was 43.1.

4.2. Scale

The continuum of the FPF is the main attribute used in this system. If the continuum is smoothly resembling the ideal case shown in Fig. 1 then this proposed system performs well. This example considers the case where the same input image varied in scale. The filters were built with the following parameters:

Filter 1 : N = 3 q_i = 100%, 90%, 80%
p = 0.8 c = { 1, 1, 1 }
Filter 2 : N = 6 q_i = 100%, 93%, 86%, 79%, 72%, 65%
p = 1.3 c = { 1, 1, 1, 1, 1, 1 }

The filter performance is shown in Fig. 5. Again the continuum is quite predictable and estimation of the scale parameter q is quite possible. Estimation of an input with unknown is trivial once the smooth curves are generated, and this estimation follows the procedure of the previous example.

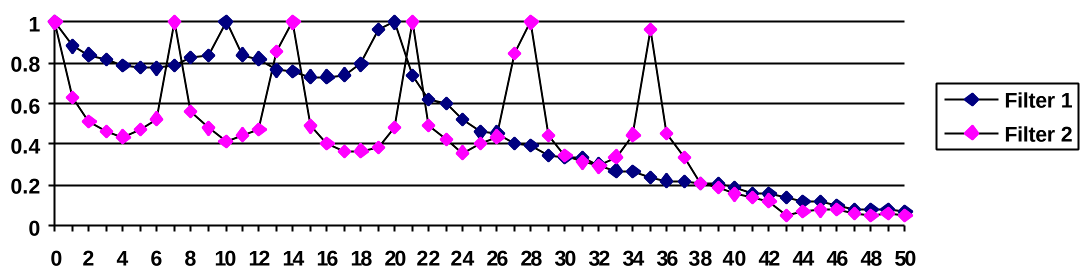

Figure 5. A graph of the continuum for the scale varying case. Measurements were taken in increments of 1% change in scale. Thus, q=1 indicates 100% scale, q=2 indicates 99% scale, etc.

4.3. Scale, Rotation and Noise

The final case allows q to represent both scale and rotation changes. The two filters used the parameters in the previous two examples to create the filters. The filter responses are shown in Fig. 6. Again the filter responses are quite smooth and predictable.

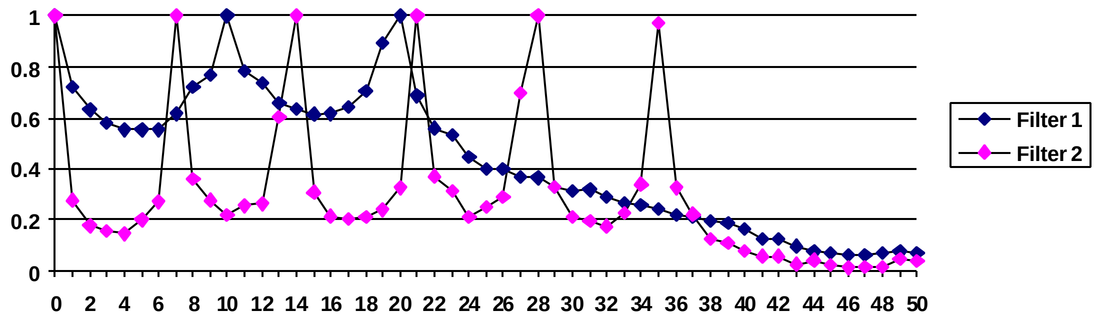

Figure 6. A graph of the continuum for the coupled scale-rotation case. The increments of rotation and scale were the same as the previous examples.

The filters were trained on noiseless images. The filters were then tested with noisy versions of the scaled-rotated input. Three tests were ran using different levels of noise. The noise allowed the original value of a pixel to vary up to 10%, 20% or 30% for the respective tests. The continuum for each of the noise tests were created and subtracted from the original continuum. Then the average and standard deviation for the absolute values of differences between the continuums was computed. Table 2 shows the results for the three noise tests for each filter.

Table 2.

	10% Filter 1	10% Filter 2	20% Filter 1	20% Filter 2	30% Filter 1	30% Filter 2
Average	0.0035	0.0098	0.0037	0.017	0.0066	0.032
Std. Dev.	0.0098	0.0081	0.0027	0.013	0.0048	0.027
Max	0.072	0.037	0.012	0.054	0.023	0.11

The differences were low in all cases in the presence of noise. The reason for this is that the FPF performs well in the presence of noise under certain conditions. The FPF trades generalization (small p) with discrimination (large p). A good generalizing filter can produce reasonable results in the presence of noise. Thus, in noisy environments the value of p may be lowered to compensate for noise. As seen in the table the FPF with the smaller p performed better in the presence of noise. It is should be noted that this tradeoff of generalization versus discrimination applies only to images with significant lower frequency content which is quite common.

These FPFs are also smooth and predictable even in the presence of noise. Once the FPF curves are known the prediction of q becomes trivial and follows the procedure shown in the first example.

5. SUMMARY

This paper presented a dual FPF method to estimate the value of a varying parameter q, where q is scale, rotation, etc. Two FPFs are constructed using vastly different parameters. An input with an unknown q is correlated with the two filters. The resultant peak values of each filter produced respective sets of possible q values. The intersection of these sets produced a single valid element that is the estimation of q. Examples where q is rotation, q is scale variances, and q is coupled scale-rotation are shown. The final example consider noisy inputs where the noise could differ the original value by 30%.

Symbols

c	The FPF constraint vector.
D	A matrix that is determined in the FPF process
h	The filter created by the FPF process.
I	An image
K	The number of elements in an image.
N	The number of images in **X**
p	The fractional power used in creating the FPF
q	a parameter to vary (i.e., scale, rotation, etc.)
Q	The set of possible q values for a specified problem.
$\hat{V}$	A matrix created from a set of vectors $\{v_1, v_2, ..., v_N\}$
X_i	An image from the set **X**
X	The set of images.
Y	**X** modified by the elements in D.

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