BINF702 FALL 2008

Chapter 7 – Hypothesis Testing: One-Sample Inference

Section 7.9 – One-Sample χ^2 Test for the Variance of a Normal Distribution

- **Eq. 7.40 (One-Sample χ^2 Test for the Variance of a Normal Distribution (Two-Sided Alternative) We compute the test statistic $X^2 = (n-1)s^2/\sigma^2_0$**
- **The rejection region is given by**

$$X^2 < \chi^2_{n-1,\alpha/2} \text{ or } X^2 > \chi^2_{n-1,1-\alpha/2}$$

- **The acceptance region is given by**

$$\chi^2_{n-1,\alpha/2} \le X^2 \le \chi^2_{n-1,1-\alpha/2}$$

Section 7.9 – One-Sample χ^2 Test for the Variance of a Normal Distribution

- **Eq. 7.41 (p-Value for a One-Sample χ^2 Test for the Variance of a Normal Distribution (Two-Sided Alternative)**
- **Test Statistic**

$$X^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

$$\text{Case I } s^2 \le \sigma_0^2 \text{ p-value given by } 2*\Pr(Y < X \mid Y \sim \chi_{n-1}^2)$$

$$\text{Case II } s^2 > \sigma_0^2 \text{ p-value given by } 2*\Pr(Y > X \mid Y \sim \chi_{n-1}^2)$$

Section 7.9 – One-Sample χ^2 Test for the Variance of a Normal Distribution

- **Ex. 7.46**
- **What is s^2 and the p-value?**

7.10 One-Sample Test for a Binomial Distribution

- **Equation 7.42 (One-Sample Test for a Binomial Proportion – Normal-Theory Method (Two-Sided Alternative) Let the test statistic be given by**

$$z = \frac{(\hat{p} - p_0)}{\sqrt{p_0 q_0 / n}}$$

The rejection region is given by $z < z_{\alpha/2}$ or $z >_{1-\alpha/2}$

The acceptence region is given by $z_{\alpha/2} \le z \le z_{1-\alpha/2}$

The test should only be used if $np_0 q_0 \ge 5$

7.10 One-Sample Test for a Binomial Distribution

- **Equation 7.43 Computation of the p-Value for the One-Sample Binomial Test – Normal-Theory Method (Two-Sided Alternative) Let the test statistic be given by**

$$z = \frac{(\hat{p} - p_0)}{\sqrt{p_0 q_0 / n}}$$

$$\text{If } \hat{p} < p_0 \text{ p-value}=2\phi(z)$$

$$\text{If } \hat{p} \geq p_0 \text{ p-value} = 2\left[1-\phi(z)\right]$$

Approximate Binomial Testing in R

```
prop.test(x, n, p = NULL,
               alternative = c("two.sided",
   "less","greater"),
               conf.level = 0.95, correct = TRUE)
```

- Problem 7.10

7.10 One-Sample Test for a Binomial Distribution

- **Eq. 7.44 (Computation of the p-Value for the One-Sample Binomial Test-Exact Method(Two-Sided Alternative)**

$$\text{If } \hat{p} \le p_0,\ p = 2P(X \le x) = \min\left[2\sum_{k=0}^{x}\binom{n}{k} p_0^k (1-p_0)^{n-k}, 1\right]$$

$$\text{If } \hat{p} > p_0,\ p = 2P(X \ge x) = \min\left[2\sum_{k=x}^{n}\binom{n}{k} p_0^k (1-p_0)^{n-k}, 1\right]$$

7.10 One-Sample Test for a Binomial Distribution (Exact Method)

```
binom.test(x, n, p = 0.5,
               alternative = c("two.sided", "less",
  "greater"), conf.level = 0.95)
```

- **Ex. 7.49**

7.10 One-Sample Test for a Binomial Distribution (Exact Method)

- **We note that this is different from the value in the text or the value using the equation in the text.**
- **How do you compute this using R?**

7.10 One-Sample Test for a Binomial Distribution

- **Eq. 7.45 (Power for the One-Sample Binomial Test (Two-Sided Alternative)) The power of the one-sample binomial test for the hypothesis H_0: p = p_0 vs. H_1: p !=p_0 for the specific alternative p = p_1 is given by**

$$\phi\left[\sqrt{\frac{p_0 q_0}{p_1 q_1}}\left(z_{\alpha/2}+\frac{|p_0-p_1|\sqrt{n}}{\sqrt{p_0 q_0}}\right)\right] \text{ where } np_0 q_0 \geq 5$$

7.10 One-Sample Test for a Binomial Distribution

- **Equation 7.46 (Sample-Size Estimation for the One-Sample Test (Two-Sided Alternative)) – Suppose we wish to test H_0: p = p_0 versus H_1: p != p_0. The sample size needed to conduct a two-sided test with significance level α and power $1 - \beta$ versus the specified alternative hypothesis p = p_1 is**

$$n = \frac{p_0 q_0 \left(z_{1-\alpha/2} + z_{1-\beta} \sqrt{\frac{p_1 q_1}{p_0 q_0}} \right)^2}{\left(p_1 - p_0 \right)^2}$$

7.10 One-Sample Test for a Binomial Distribution

- **Can you write a function to implement Equation 7.46?**

7.10 One-Sample Test for a Binomial Distribution

- **Our function in action on Ex. 7.51**

Section 7.11 – One-Sample Inference for the Poisson Distribution

- **Eq. 7.47 (One –Sample Inference for the Poisson Distribution (Small-Sample Test –Critical Value Method) Let X be a Poisson random variable with expected value = μ. To test the hypothesis H_0: $\mu = \mu_0$ versus H_1: $\mu != \mu_0$ using a two-sided test with significance level α,**

1. **Obtain the two-sided 100% x $(1-\alpha)$ confidence interval for μ based on the observed value x of X. Denote the confidence interval (c_1, c_2)**
2. **If $\mu_0 < c_1$ or $\mu_0 > c_2$, then reject H_0, if $c_1 <= \mu_0 <= c_2$, then accept H_0.**

Section 7.11 – One-Sample Inference for the Poisson Distribution

- **Eq. 7.48 (One-Sample Inference for the Poisson Distribution (Small-Sample Test p-Value Method) Let μ = expected value of a Poisson distribution. To test the hypothesis H_0: $\mu = \mu_0$ versus H_1: $\mu \mathrel{!=} \mu_0$,**

1. **Compute x = observed number of deaths in the study population**
2. **Under H_0, the random variable X will follow a Poisson distribution with parameter μ_0. Thus the two-sided p-value is given by**

$$\min\left(2\sum_{k=0}^{x}\frac{e^{-\mu_0}\mu_0^k}{k!},1\right) \text{ if } x < \mu_0$$

$$\min\left(2\left(1-\sum_{k=0}^{x}\frac{e^{-\mu_0}\mu_0^k}{k!}\right),1\right) \text{ if } x \geq \mu_0$$

Section 7.11 – One-Sample Inference for the Poisson Distribution

- **Ex. 7.54**
- **Can you solve this using R as a calculator?**

Section 7.11 – One-Sample Inference for the Poisson Distribution

- **Def. 7.16 – The <u>standardized mortality ratio</u> (SMR) is define by 100% x O/E = 100% x the observed number of deaths in the study population divided by the expected number of deaths in the study population under the assumption that the mortality rates for the study population are the same as those for the general population. For nonfatal conditions the standardized mortality ratio is sometimes known as the <u>standardized morbidity ratio</u>.**

Section 7.11 – One-Sample Inference for the Poisson Distribution

- **Eq. 7.49 (One-Sample Inference for the Poisson Distribution (Large-Sample Test) Let μ = expected value of a Poisson random variable. To test the hypothesis H_0: $\mu = \mu_0$ versus H_1: μ != μ_0**

(1) Compute x = observed number of events in the study population

(2) Compute the test statistic

$$X^2 = \frac{(x-\mu_0)^2}{\mu_0} = \mu_0\left(\frac{SMR}{100}-1\right)^2 \sim \chi_1^2 \text{under } H_0$$

(3) For a two-sided test at level α, H_0 is rejected if

$$X^2 > \chi^2_{1,1-\alpha}$$

and H_0 is accepted if $X^2 \le \chi^2_{1,1-\alpha}$

Section 7.11 – One-Sample Inference for the Poisson Distribution

- **Eq. 7.49 (continued)**

(4) The exact p-value is given by $\Pr(\chi_1^2 > X^2)$

(5) This test should only be used if $\mu_0 \geq 10$

(6) In addition, an approximate 100% x (1-α) CI for μ is given by

$$x \pm z_{1-\alpha/2}\sqrt{x}$$

Homework

- **Set # 1**
  - **1,2,3,4,7,8,9,26,27,28**

- **Set #2**
  - **10,11,12, 13, 14, 19, 20, 22, 29, 30, 31, 53, 54, 55, 73, 74, 75**